Parametric Study of Mechanical Behavior of Superconducting Solenoids

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Contents

Introduction

Superconducting Solenoids in high magnetic fields are heavily loaded.

It is important to calculate the mechanical behavior which depends on a number of parameters.

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A fully analytical model is being realized to perform parametric studies on different configurations of solenoids.

Accomplished tasks

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- Verify previous analytical model
  [E. Terzini and E. Barzi, "Analytical Study of Stress State in HTS Solenoids", FERMILAB-TM-2448-TD]
- Understand and run Ansys mesomechanical model
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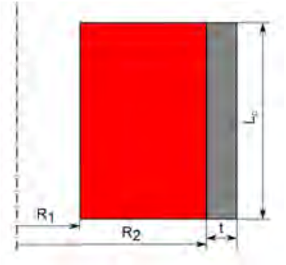
- Sensitivity analysis of all parameters (E, ν, μ_0)
- Study of reinforced coils with multiple skins

Contents

Analytical model

Lamé's equation

$$\frac{E}{1-\nu^2}\left(\frac{d^2u}{dr^2} - \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2}\right) + f = 0$$



Boundary conditions

$$\begin{aligned}
\sigma_{rr,c}(R_1) &= 0\\
\sigma_{rr,s}(R_2+t) &= 0\\
\sigma_{rr,c}(R_2) - \sigma_{rr,s}(R_2) &= 0\\
u_{c_2}(R_2) - u_{s_2}(R_2) &= 0
\end{aligned}$$

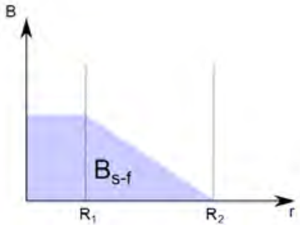
Magnetic model

Self-field

$$B_0(\alpha,\beta) = R_1\mu_0 J\beta ln\left(\frac{\sqrt{\alpha^2+\beta^2}+\alpha}{\sqrt{1+\beta^2}+1}\right)$$

where $\alpha = \frac{R_2}{R_1}$ and $\beta = \frac{L_c}{2}\frac{1}{R_1}$

$$B_0(r) = \begin{cases} B_0 & if\ r < R_1 \\ B_0\left(1-\frac{r-R_1}{R_2-R_1}\right) & if\ r \geq R_1 \end{cases}$$



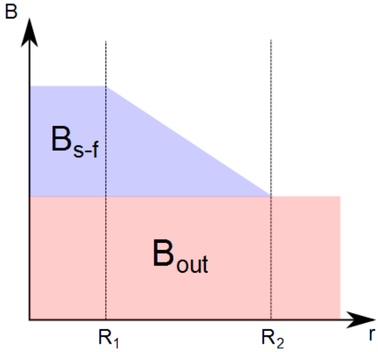
$$f = \frac{1-\nu^2}{E}J(B_0)\frac{B_0}{R_2-R_1}\left(\frac{R_2 r^2}{3}-\frac{r^3}{8}\right)$$

Magnetic model

Insert coil

Superposition of self-field and outer field

$$B_{out}(r) = cost.$$



$$f = \frac{1-\nu^2}{E} J(B_{tot}) \left[\frac{B_0}{R_2 - R_1} \left(\frac{R_2 r^2}{3} - \frac{r^3}{8} \right) + B_{out} \frac{r^2}{3} \right]$$

Effect of anisotropy

Material	E (GPa)	area (mm^2)
YBCO	110	0.4
Kapton	5.5	0.1125
Epoxy	4.5	0.05

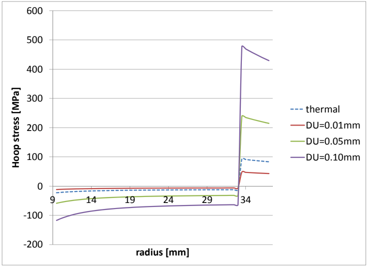
An averaged Young modulus depending on the ratio among the areas was introduced.

$$E_{av} = 79.7 GPa$$

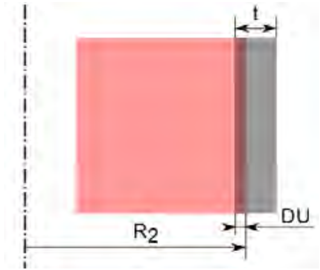
Temperature ∼ Interference

Thermal deformations

$$\begin{pmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{pmatrix} = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu \\ \nu & 1 \end{pmatrix} \begin{pmatrix} \epsilon_{rr} - \alpha\Delta T \\ \epsilon_{\theta\theta} - \alpha\Delta T \end{pmatrix}$$



Assembly interference



Configuration analyzed

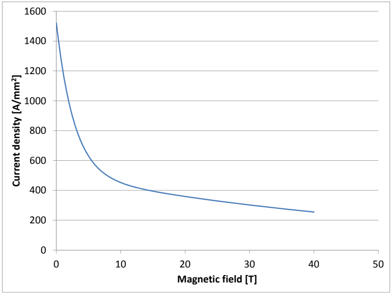
Geometry

$$R_1 = 9.5mm$$
$$R_2 = 31mm$$
$$L_c = 126mm$$
$$t = 4mm$$

Magnetic properties

$$B = 13.4T$$
$$J(B) = 522.7\frac{A}{mm^2}$$

Engineering Current Density



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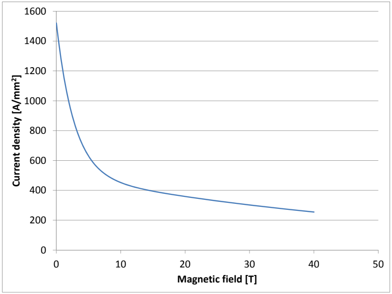
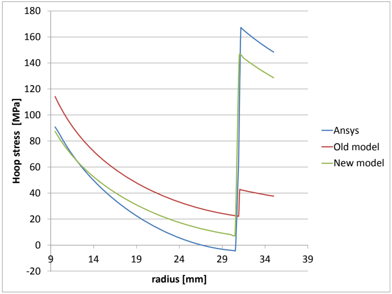


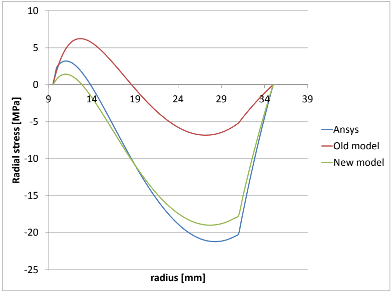
All the stress distributions are in the mid-plane

Results - Self-field

Hoop stresses



Radial stresses



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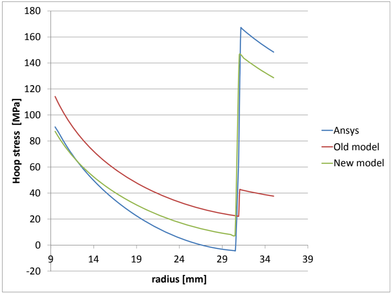

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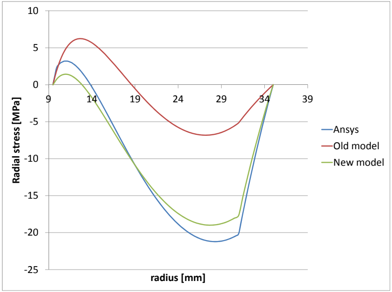
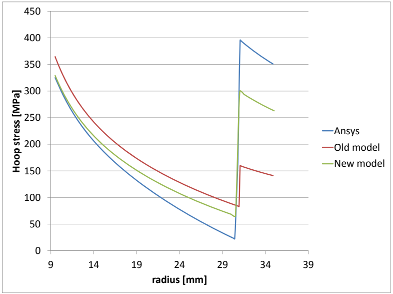


$$e_{max} = 12\%$$

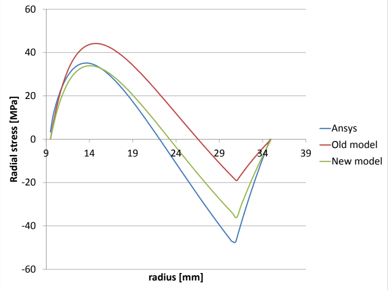
Results - Insert coil

10 T background field

Hoop stresses



Radial stresses



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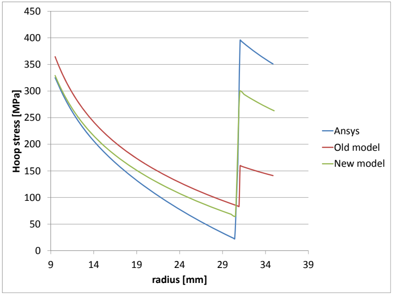

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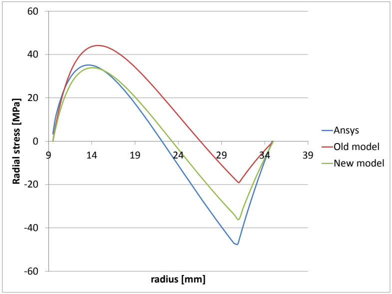
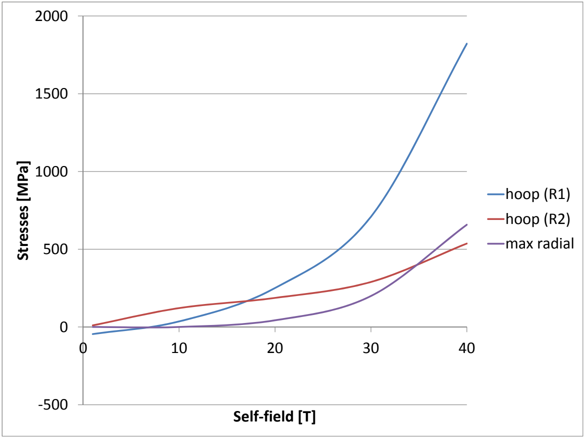


$$e_{max} = 23\%$$

Contents

Max stresses (self-field)



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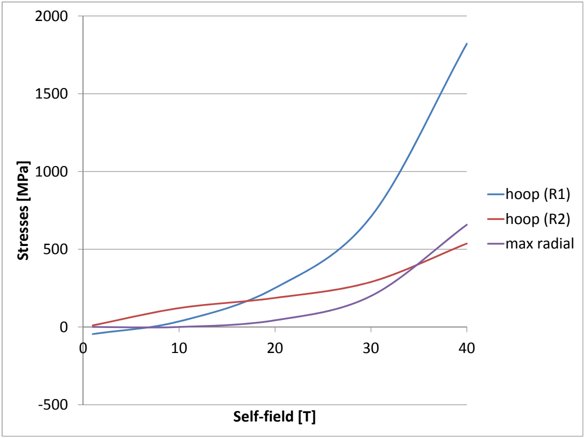
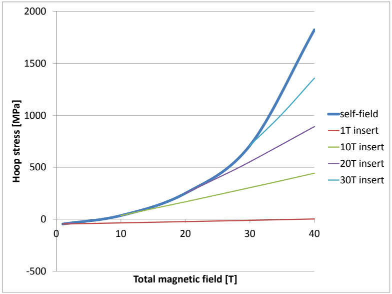


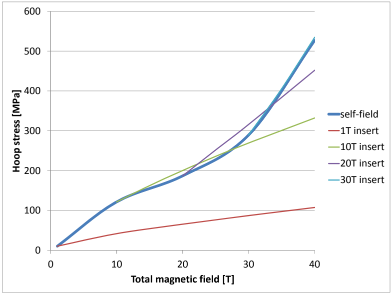
At high fields the coil is the most critical

Max stresses (insert-coil)

Hoop at R_1 (COIL)

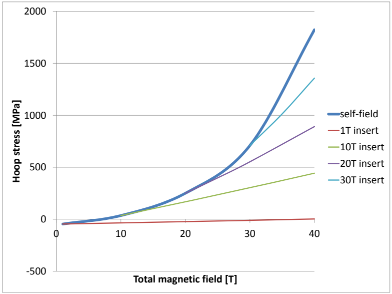


Hoop at R_2 (SKIN)

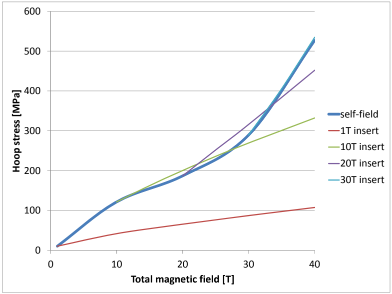


Max stresses (insert-coil)

Hoop at R_1 (COIL)



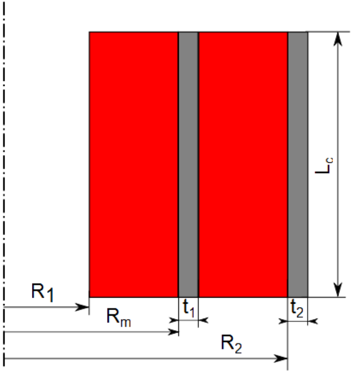
Hoop at R_2 (SKIN)



Smaller solenoids have smaller stresses

Contents

Double skin configuration

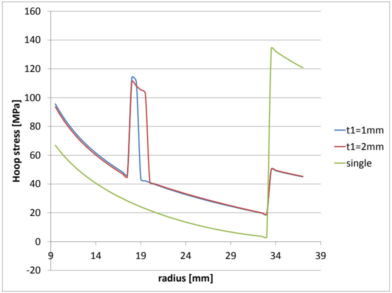


Boundary conditions

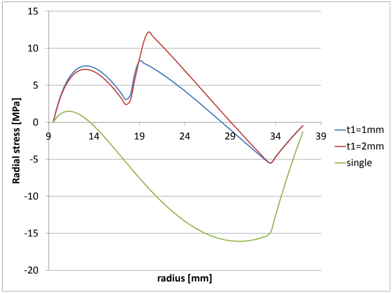
$$\sigma_{rr,c_1}(R_1) = 0$$
$$\sigma_{rr,s_2}(R_2 + t_2) = 0$$
$$\sigma_{rr,c_1}(R_m) - \sigma_{rr,s_1}(R_m) = 0$$
$$\sigma_{rr,s_1}(R_m + t_1) - \sigma_{rr,c_2}(R_m + t_1) = 0$$
$$\sigma_{rr,c_2}(R_2) - \sigma_{rr,s_2}(R_2) = 0$$
$$u_{c_1}(R_m) - u_{s_1}(R_m) = 0$$
$$u_{s_1}(R_m + t_1) - u_{c_2}(R_m + t_1) = 0$$
$$u_{c_2}(R_2) - u_{s_2}(R_2) = 0$$

Effect of steel thickness

Hoop stress



Radial stress



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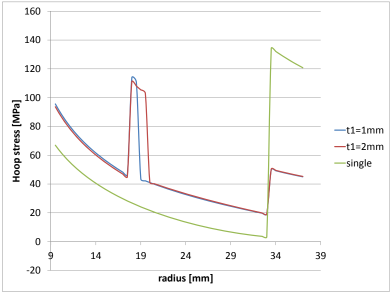
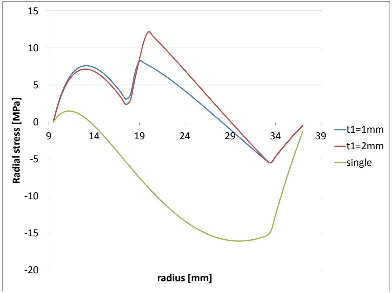


Radial stress



Max stresses can be reduced

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Future developments

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- Deepen multiple coil configurations [Hahn (MIT)2011]

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- Optimize geometrical parameters
- Deepen multiple coil configurations [Hahn (MIT)2011]
- Realize technical applications of the analytical model

Axial stresses

Ansys model

- Glued connection imposed
- Over estimation of the stresses

Analytical model

- Impose a constant axial deformation at the mid plane
- Consider the effect of the radial component of the magnetic field

$$B_r = \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{(a+r)^2 + z^2}} \left[K(k) + \frac{a^2 - r^2 - z^2}{(a-r)^2 + z^2} E(k) \right]$$